

# Review of Physics Concepts

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Mechanics

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Physics is the most basic of all the sciences. It deals with the behavior and structure of matter.

## Motion in One Dimension: Kinematics

Average Velocity:  $\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$

Instantaneous Velocity:  $v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

Average Acceleration:  $\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$

Instantaneous Acceleration:  $a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

Equations for Motion with Constant Acceleration:

<u>Equation</u>	<u>Missing Quantity</u>
$v = v_0 + at$	$x - x_0$
$x - x_0 = v_0 t + \frac{1}{2} at^2$	$v$
$v^2 = v_0^2 + 2a(x - x_0)$	$t$
$x - x_0 = \frac{1}{2}(v_0 + v)t$	$a$
$x - x_0 = vt - \frac{1}{2} at^2$	$v_0$

Acceleration due to gravity:

$$g = 9.8 \text{ m/s}^2 = 32 \text{ ft/s}^2$$

## Newton's Three Laws of Motion

### • Newton's First Law of Motion - Law of Inertia

Every object continues in its state of rest, or of motion in a straight line at constant speed, unless it is compelled to change that state by forces exerted upon it.

### • Newton's Second Law of Motion

The acceleration produced by a net force on an object is directly proportional to the magnitude of the net force, is in the same direction as the net force, and is inversely proportional to the mass of the body.

$$a \propto \frac{F}{m} \quad \text{or} \quad F = ma$$

### • Newton's Third Law of Motion - Action and Reaction

Whenever one object exerts a force on a second object, the second exerts an equal and opposite force on the first.

Weight: An important force acting on any body is its weight  $W$ , defined as the force

$$W = mg$$

$$1 \text{ lb} = 4.45 \text{ N}$$

Weight is a vector measured in force units (N), whereas mass is a scalar whose SI Unit is the kilogram (kg).

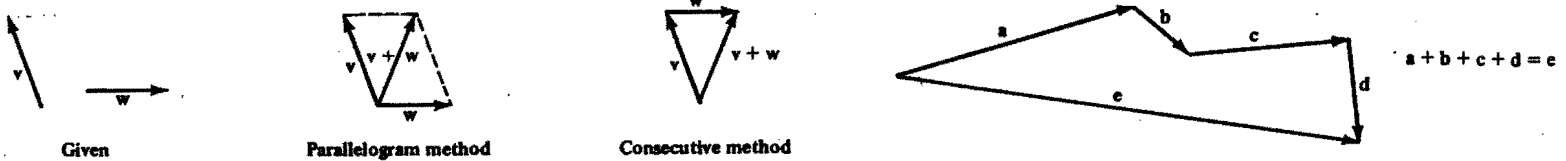
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# Vectors

## Scalars and Vectors

Scalars have magnitude only. They are specified by a number with a unit and obey the rules of ordinary algebra. Vectors, such as displacement, have both magnitude and direction and obey the rules of vector algebra.

## Addition of Vectors:



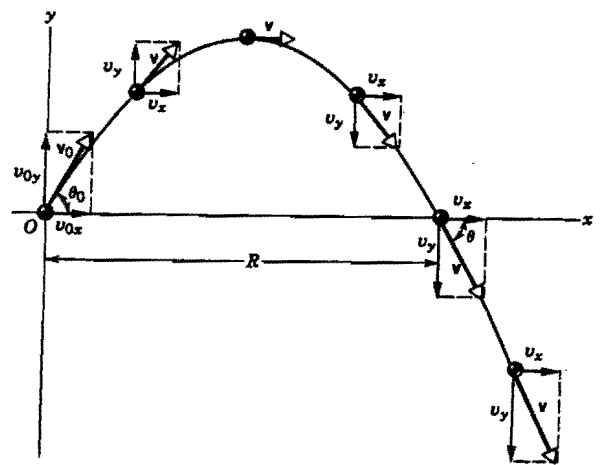
We call  $v+w$  the vector sum, or resultant.

Vectors can be subtracted as well as added. If  $v$  and  $w$  are two vectors, then  $v-w$  means  $v+(-w)$ .

## Motion in a Plane

Projectile Motion: For a launch angle  $\theta_0$  and initial position at the origin of coordinates, the motion is described by the equations:

$$\begin{aligned}x - x_0 &= (v_0 \cos \theta_0) t \\y - y_0 &= (v_0 \sin \theta_0) t - \frac{1}{2} g t^2 \\v_y &= v_0 \sin \theta_0 - g t \\v_y^2 &= (v_0 \sin \theta_0)^2 - 2g(y - y_0)\end{aligned}$$



The Trajectory: is a parabola whose equation is:

$$y = (\tan \theta_0) x - \left( \frac{g}{2(v_0 \cos \theta_0)^2} \right) x^2$$

and the projectile's range (the horizontal distance between launch and its return to the launching height) is:

$$R = \frac{v_0^2}{g} \sin 2\theta_0$$

## Frictional Force

The force between surfaces in relative motion is called sliding friction.

Coefficient of Kinetic (sliding) Friction ( $\mu_k$ ):  $f_k = \mu_k N$

If two objects are not in relative motion, static friction is the force that opposes the start of motion.

Coefficient of Static Friction ( $\mu_s$ ):  $f_s = \mu_s N$

## Circular Motion: Centripetal & Centrifugal Force

### Uniform Circular Motion

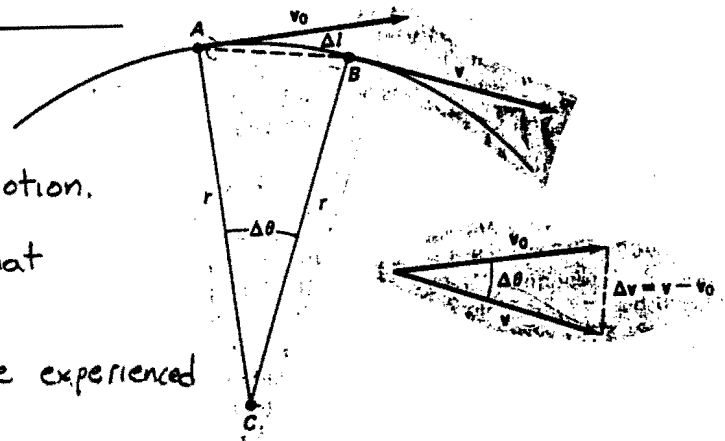
When a body moves with constant speed in a circle, it is said to have uniform circular motion.

**Centripetal Force:** A center-seeking force that causes an object to follow a circular path.

**Centrifugal Force:** An apparent outward force experienced by a rotating body.

**Centripetal Acceleration:**  $a_c = \frac{v^2}{r}$        $v = \frac{2\pi r}{T}$

**Centripetal Force:**  $F_c = ma_c = m\left(\frac{v^2}{r}\right)$



## Rotational Mechanics (Torque); Equilibrium

Torque  $\tau$  is a measure of the "turning" action of a force  $F$ .

$$\tau = rF \sin \phi = r_{\perp} F$$

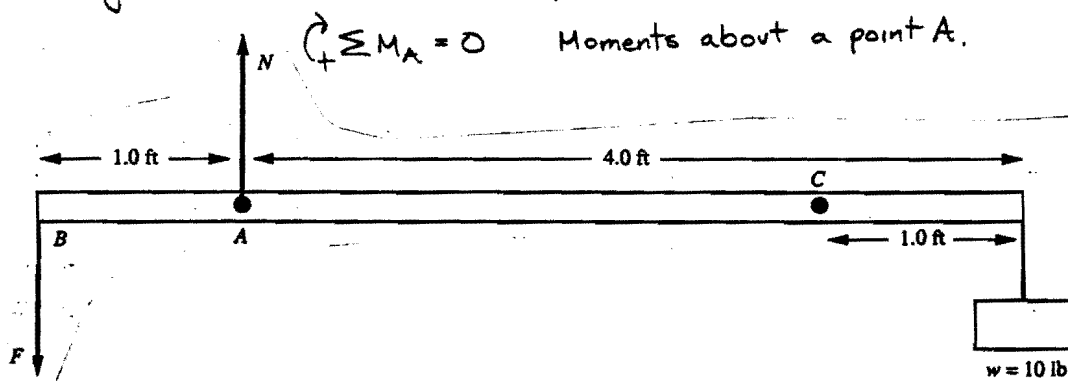
Balance of Torques:  $\sum \tau_x = 0$   
 $\sum \tau_y = 0$

**First Condition of Equilibrium:** For a body to be in equilibrium the first condition that must be met is that the resultant force acting on the body must be zero.

$$\uparrow \sum F_x = 0$$

$$\uparrow \sum F_y = 0$$

**Second Condition of Equilibrium:** If a body is in equilibrium, not only must the forces acting on it add up to zero but also all torques exerted by these forces with respect to any axis must also add up to zero.



A system is balanced when the two torques are equal:

counterclockwise torque = clockwise torque

$$(F_{\perp} d)_{ccw} = (F_{\perp} d)_{cw}$$

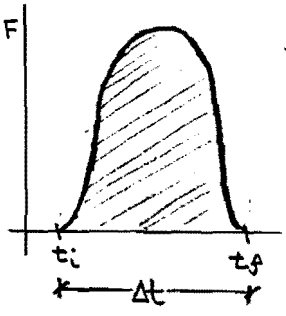
# Momentum : Collisions in One Dimension

The linear momentum of a body is defined as the product of its mass and velocity.

$$p = mv$$

$p$  = momentum (kg m/s)  
 $m$  = mass (kg)  
 $v$  = velocity (m/s)

Impulse :  $F$



$$I = \int_{t_i}^{t_f} F dt$$

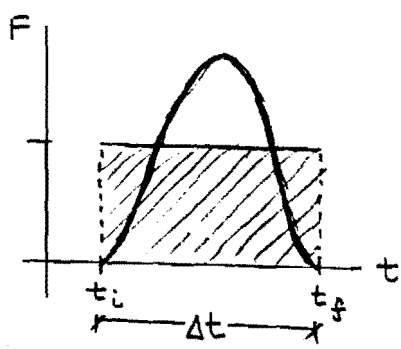
Newton's 2nd:  $F = dp/dt$

$$I = \int_{t_i}^{t_f} F dt = \int_{t_i}^{t_f} dp/dt dt = p_f - p_i = \Delta p$$

$$F \Delta t = \Delta p = mv_f - mv_i = m(v_f - v_i)$$

Time Average of a Force :

$$F_{av} = \frac{1}{\Delta t} \int_{t_i}^{t_f} F dt = \frac{I}{\Delta t} \therefore F_{av} = \frac{I}{\Delta p} \text{ or } \frac{\Delta p}{\Delta t}$$



Conservation of Momentum :

momentum before = momentum after

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$p_1 + p_2 = p_1' + p_2'$$

Law:

- (1) The total momentum of an isolated system of bodies remains constant.
- (2) In the absence of an external force, the momentum of a system remains unchanged.

Coefficient of Restitution :

$$e = \frac{v_2 - v_1}{u_2 - u_1} \quad , \quad \begin{array}{l} v_1 \& \dot{\;} v_2 \text{ velocity after impact} \\ u_1 \& \dot{\;} u_2 \text{ velocity before impact} \end{array}$$

- Perfect elastic collision,  $e = 1$
- Completely inelastic collision,  $e = 0$  stick together

$$v_2' - v_1' = -e(v_2 - v_1)$$

Elastic Collisions - When objects collide without lasting deformation or the generation of heat.

Inelastic Collisions - Whenever colliding objects become tangled or couple together.

Impact refers to a force (N)

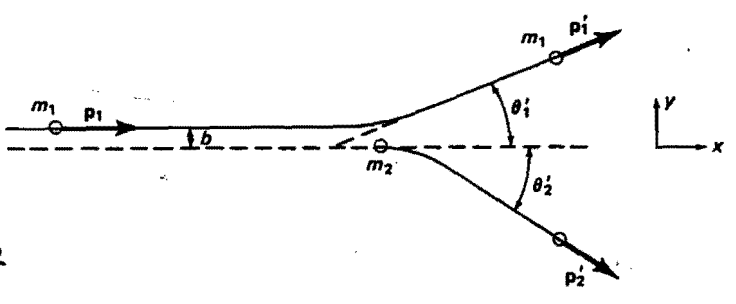
Impulse is impact force  $\times$  time (Ns)

## Collisions in Two Dimensions

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

$$m_1 v_1 = m_1 v_1' \cos \theta_1 + m_2 v_2' \cos \theta_2$$

$$0 = m_1 v_1' \sin \theta_1 + m_2 v_2' \sin \theta_2$$



Considering  $v_2' = 0$

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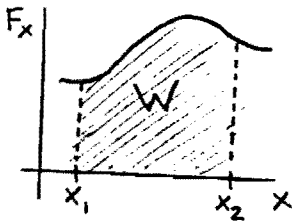
## Work and Energy

The amount of work done is the scalar quantity calculated from

$$W = Fd \cos \phi = F \cdot d$$

Units:  $1 \text{ J} = 1 \text{ N} \cdot \text{m} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 0.738 \text{ ft} \cdot \text{lb}$

Work by a Variable Force:



$$W = \int_{x_1}^{x_2} F_x dx = F_x \Delta x$$

Power: The rate at which a force does work is called the power input  $P$  of the force.

$$dW = F \cdot ds = F \cdot v dt$$

$$P = \frac{dW}{dt} = \frac{F dx}{dt} = F \left( \frac{dx}{dt} \right)$$

SI Unit of power:  $1 \text{ watt} = 1 \text{ W} = 1 \text{ J/s} = 0.738 \text{ ft} \cdot \text{lb/s}$

$1 \text{ hp} = 550 \text{ ft} \cdot \text{lb/s} = 746 \text{ W}$

Translational Kinetic Energy:  $W = Fd = mad = m \left( \frac{v^2}{2d} \right) d = \frac{1}{2} mv^2$   
(The energy of motion)

$$KE = \frac{1}{2} mv^2$$

Potential Energy: Energy stored and held in readiness by an object by virtue of its position. In a stored state it has the potential for doing work.

$$PE = mgy$$

Work Energy Theorem:

$$W = \int_a^b \frac{d}{dt} \left( \frac{1}{2} m [v(t)]^2 \right) dt = \frac{1}{2} m [v(b)]^2 - \frac{1}{2} m [v(a)]^2$$

$$W = KE' - KE = \Delta KE$$

$$W = \Delta KE = -\Delta PE$$

Law of Conservation of Energy:

Energy cannot be created or destroyed. It may be transformed from one form in another, but the total amount of energy never changes.

$$KE + PE = KE' + PE'$$

$$\frac{1}{2} mv^2 + mgy = \frac{1}{2} mv'^2 + mgy'$$

## Universal Gravitation

Kepler's Three Laws of Planetary Motion:

1. Each planet moves in an elliptical orbit, with the sun at one focus of the ellipse.
2. The radius vector from the sun to the moving planet sweeps out area at a constant rate.
3. The square of the time required for each planet to travel once around its elliptical orbit is proportional to the cube of the length of the semimajor axis of the ellipse.

$$\left(\frac{T_a}{T_b}\right)^2 = \left(\frac{r_a}{r_b}\right)^3$$

$T_a$  &  $T_b$  are their periods

$r_a$  &  $r_b$  are their average distances

Newton's Law of Gravitation:

Any pair of particles in the universe attract each other with a force whose magnitude is

$$F = G \frac{m_1 m_2}{r^2}$$

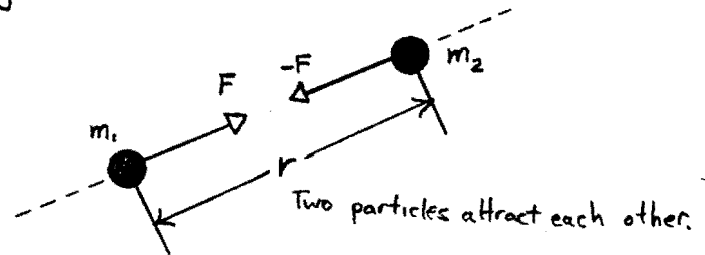
$$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$$

$m_1$  &  $m_2$  are expressed in kilograms, and  $r$  in meters.

Acceleration due to gravity:  $g = \frac{GM}{r^2}$

Velocity of a satellite:  $v = \sqrt{\frac{GM}{r}}$

The Law of Periods:  $T = 2\pi \sqrt{\frac{r^3}{GM}}$



## The Fluid States

Density: The density of any material is defined as its mass per unit volume.

$$\rho = \frac{\Delta m}{\Delta V}$$

Fluid Pressure: Pressure is defined as the force on a unit surface area.

$$P = \frac{F}{A}$$

SI Unit of pressure:  $1 \text{ Pa} = 1 \text{ N/m}^2$

$1 \text{ atm} = 1.01 \times 10^5 \text{ Pa} = 760 \text{ torr} = 14.7 \text{ lb/in}^2$

### Pascals Principle

A change in the pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and to the walls of the containing vessel.

### Archimedes Principle

A body wholly or partially immersed in a fluid will be buoyed up by a force equal to the weight of the fluid that it displaces.

### Bernoulli's Principle

If the speed of a fluid particle increases as it travels along a streamline, the pressure of the fluid must decrease, and conversely.

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