Analytic Geometry: Parabolas, Ellipses, and Hyperbolas

Tuesday, April 22, 1997

Parabolas

- A *parabola* is the set of all points in a plane that are equidistant from a fixed point called the *focus* and a line called the *directrix*.

The *axis* of a parabola is the line that passes through the focus and is perpendicular to the directrix. A parabola is symmetric about its axis.

The length of the *latus rectum* is 4a. The enpoints of the latus rectum are given by (h - 2a, k + a) and (h + 2a, k + a).

a is the distance from the vertex to the focus and also the distance from the vertex to the directrix. The latus rectum and the directrix are also parallel to each other. The parabola never crosses the directrix.

Parabola with Horizontal Directrix (up and down)

General Form: $Ax^2 + Dx + Ey + F = 0$

Standard Form: Center at (h, k)

$(x-h)^2 = 4a(y-k)$ when it opens upward

$(x-h)^2 = -4a(y-k)$ when it opens downward

Vertex at the origin, focus at (0, a): $x^2 = 4ay$ when it opens upward

Vertex at the origin, focus at (0, -a): $x^2 = -4ay$ when it opens downward

Parabola with Vertical Directrix (left and right)

General Form: $Cy^2 + Dx + Ey + F = 0$

Standard Form: Center at (h, k)

$(y-k)^2 = 4a(x-h)$ when it opens to the right

$(y-k)^2 = -4a(x-h)$ when it opens to the left

Vertex at origin, focus at (a, 0) $y^2 = 4ax$ when it opens to the right

Vertex at origin, focus at (-a, 0) $y^2 = -4ax$ when it opens to the left

Robbins (1998)

Analytic Geometry: Parabolas, Ellipses, and Hyperbolas

Tuesday, April 22, 1997

Ellipses

- An *ellipse* is the set of all points in a plane the sum of whose distances from two fixed points is a constant. Each of the two fixed points is called a focus of the ellipse. (foci)

The point halfway between the foci of an ellipse is called the *center* of the ellipse. The line segment through the foci that terminates on the ellipse is called the *major axis*. The line segment through the center, perpendicular to the major axis, and terminating on the ellipse is called the *minor axis*. The ends of the major axis are called the *vertices* of the ellipse and the ends of the minor axis are called the *covertices*. Each of the line segments passing through the foci, perpendicular to the major axis, and terminating on the ellipse is called a *latus rectum*. (plural: latus recta)

Ellipse with Horizontal Major Axis

General Form: $Ax^2 + Cy^2 + Dx + Ey + F = 0$

Standard Form: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ Center at origin: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Vertices: V1(h - a, k) and V2(h + a, k) Length of latus recta = $\frac{2b^2}{a}$

Covertices: CV1(h, k - b) and CV2(h, k + b) Length of major axis = 2a

Foci: F1(h - c, k) and F2(h + c, k), where $c^2 = a^2 - b^2$ Length of minor axis = 2b

Ellipse with Vertical Major Axis

General Form: $Ax^2 + Cy^2 + Dx + Ey + F = 0$

Standard Form: $\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$ Center at origin: $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$

Vertices: V1(h, k - a) and V2(h, k + a) Length of latus recta = $\frac{2b^2}{a}$

Covertices: CV1(h - b, k) and CV2(h + b, k) Length of major axis = 2a

Foci: F1(h, k - c) and F2(h, k + c), where $c^2 = a^2 - b^2$ Length of minor axis = 2b

Vertex - Center = a
Covertex - Center = b
Foci - Center = c

Eccentricity: $e = \frac{\sqrt{a^2 - b^2}}{a}$

Sum of distances from any point on ellipse to the foci: $2a$

Robbins (1998)

Analytic Geometry: Parabolas, Ellipses, and Hyperbolas

Tuesday, April 22, 1997

Hyperbolas

- A hyperbola is the set of all points in a plane the difference of whose distances from two fixed points, called foci, is a constant.

The point halfway between the two foci or vertices is called the *center* of the hyperbola. The line segment between the two vertices is called the *transverse axis* of the hyperbola. The line segment passing through each focus perpendicular to the transverse axis and terminating on the hyperbola is called a *latus rectum*. A hyperbola has two linear *asymptotes*, which intersect at the center. The line segment that passes through the center of the hyperbola perpendicular to the transverse axis and that terminates on opposite sides of the rectangle formed by the asymptotes is called the *conjugate axis*.

Hyperbola with Horizontal Transverse Axis (left and right)

General Form: $Ax^2 + Cy^2 + Dx + Ey + F = 0$ Length of Latus Recta = $\frac{2b^2}{a}$

Standard Form: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ Asymptotes: $y - k = \pm\frac{b}{a}(x-h)$

Vertices: V1(h - a, k) and V2(h + a, k)

Ends of Conjugate Axis: E1(h, k - b) and E2(h, k +b)

Foci: F1(h - c, k) and F2(h + c, k), where $c^2 = a^2 + b^2$

Hyperbola with Vertical Transverse Axis (up and down)

General Form: $Ax^2 + Cy^2 + Dx + Ey + F = 0$ Length of Latus Recta = $\frac{2b^2}{a}$

Standard Form: $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ Asymptotes: $y - k = \pm\frac{a}{b}(x-h)$

Vertices: V1(h, k - a) and V2(h, k + a)

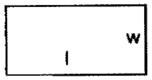
Ends of Conjugate Axis: E1(h - b, k) and E2(h + b, k)

Foci: F1(h, k - c) and F2(h, k + c), where $c^2 = a^2 + b^2$

In a hyperbola, the term which is positive contains a^2 in the denominator.

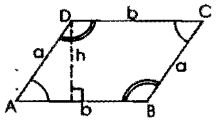
Robbins (1998)

GEOMETRY – Formulas



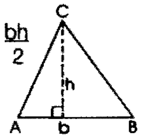
Perimeter = 2(l + w)
Area = l x w

Volume = l x w x h



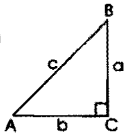
Area of ABCD=bh

Area of $\Delta ABC = \frac{bh}{2}$



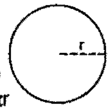
$c^2 = a^2 + b^2$ (Pythagorean theorem)

Area of $\Delta ABC = \frac{ab}{2}$



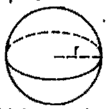
Circumference of a circle = $2\pi r$

Area of a circle = πr^2



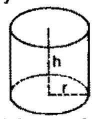
Surface area of sphere = $4\pi r^2$

Volume of a sphere = $\frac{4\pi r^3}{3}$



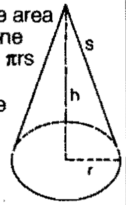
Surface area of cylinder = $2\pi rh + 2\pi r^2$

Volume of cylinder = $\pi r^2 h$

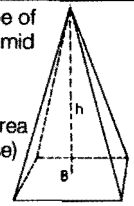


Surface area of a cone = $\pi r^2 + \pi rs$

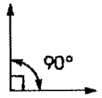
Volume of a cone = $\frac{\pi r^2 h}{3}$



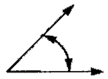
Volume of a pyramid = $\frac{Bh}{3}$ (B = area of base)



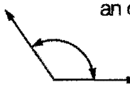
Angles and triangles:



a right angle is 90°



an acute angle is less than 90°



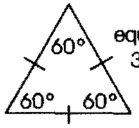
an obtuse angle is more than 90° but less than 180°

a straight angle is 180°

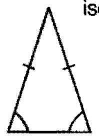
1 complete angle of rotation = 360°

two complementary angles - add up to 90°
two supplementary angles - add up to 180°

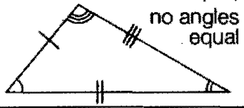
equilateral triangle
3 sides of equal length,
3 angles of 60° each



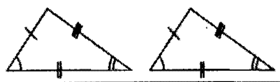
isosceles triangle
2 sides of equal length,
base angles are equal



scalene triangle
no sides equal,
no angles equal



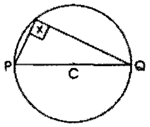
Congruency cases:



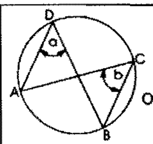
side, side, side
side, angle, side
angle, side, angle

hypotenuse, side

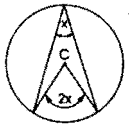
Circle theorems:



∠x = 90°
PQ is a diameter



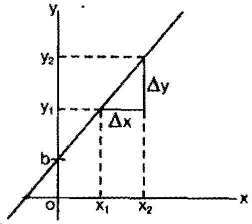
∠a = ∠b
angles subtended on the same arc AB



C is the center of the circle

TRIGONOMETRY

Slopes:



Equation of a straight line

$y - y_1 = m(x - x_1)$

where m = slope = $\frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

or

$y = mx + b$

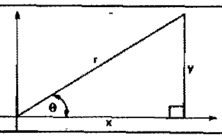
where m = slope, b = y-intercept

Trigonometric ratios:

$\sin\theta = \frac{y}{r}$ (opposite/hypotenuse)= $1/\csc\theta$

$\cos\theta = \frac{x}{r}$ (adjacent/hypotenuse)= $1/\sec\theta$

$\tan\theta = \frac{y}{x}$ (opposite/adjacent)= $1/\cot\theta$



$\tan\theta = \sin\theta/\cos\theta$

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$\cos^2\theta - \sin^2\theta = \cos 2\theta$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

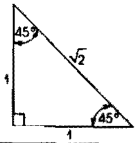
$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

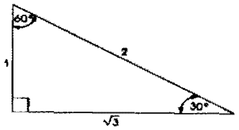
$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$



$\sin 45° = \frac{1}{\sqrt{2}}$

$\cos 45° = \frac{1}{\sqrt{2}}$

$\tan 45° = 1$



$\sin 30° = \frac{1}{2}$ $\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$ $\cos 60° = \frac{1}{2}$

$\tan 30° = \frac{1}{\sqrt{3}}$ $\tan 60° = \sqrt{3}$

CAST

Quad II sin+ | Quad I all ratios+
Quad III tan+ | Quad IV cos+

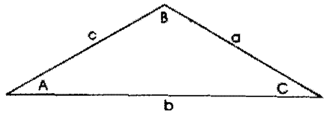
Sine Law: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Law:

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

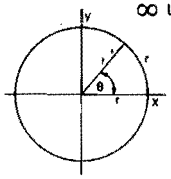
$c^2 = a^2 + b^2 - 2ab \cos C$



Value of trig ratio

θ	0	π/2	π	3π/2	2π
sin θ	0	1	0	–1	0
cos θ	1	0	–1	0	1
tan θ	0	∞	0	–∞	0

∞ undefined (infinite)



θ=1 radian

2π radians = 360°

Robbins (1998)