

Newton's Method: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ Average Value of f : (Mean Value Theorem) $\frac{1}{b-a} \int_a^b f(x) dx$ Average rate of change: $\frac{f(b) - f(a)}{b-a}$

Curve Sketching:

- Look for possible **extreme points** of $f(x)$ by setting $f'(x)=0$ and solving for x . Plug in that value into $f(x)$ to obtain exact coordinates.
- Plug in the same value of x into $f''(x)$ to obtain the **concavity**.
- Look for possible **inflection points** by setting $f''(x)=0$ and solving for x . Plug in this value of x into $f(x)$ to obtain exact coordinates.

Differentiating the Trig Functions:

$$\begin{aligned} \frac{d}{dx}(\sin u) &= \cos u \frac{du}{dx} & \frac{d}{dx}(\cot u) &= -\csc^2 u \frac{du}{dx} \\ \frac{d}{dx}(\cos u) &= -\sin u \frac{du}{dx} & \frac{d}{dx}(\sec u) &= \sec u \tan u \frac{du}{dx} \\ \frac{d}{dx}(\tan u) &= \sec^2 u \frac{du}{dx} & \frac{d}{dx}(\csc u) &= -\csc u \cot u \frac{du}{dx} \end{aligned}$$

Integrating the Trig Functions:

$$\begin{aligned} \int \cos x dx &= \sin x + c & \int \csc^2 x dx &= -\cot x + c \\ \int \sin x dx &= -\cos x + c & \int \sec x \tan x dx &= \sec x + c \\ \int \sec^2 x dx &= \tan x + c & \int \csc x \cot x dx &= -\csc x + c \\ \int \cos^2 x dx &= \frac{x}{2} + \frac{\sin 2x}{4} + c & \int \sin^2 x dx &= \frac{x}{2} - \frac{\sin 2x}{4} + c \end{aligned}$$

The Disk Method

Volume of a solid of Revolution (about the x-axis): $V = \int_a^b \pi [f(x)]^2 dx$

Volume of a solid of Revolution (about the y-axis): $V = \int_c^d \pi [f(y)]^2 dy$

The Washer Method

Washer method about x-axis: $V = \int_a^b \pi ([f(x)]^2 - [g(x)]^2) dx$

The Shell Method

Shell method about x-axis: $V = \int_c^d 2\pi y [F(y) - G(y)] dy$

Washer method about y-axis: $V = \int_c^d \pi ([F(y)]^2 - [G(y)]^2) dy$

Shell method about y-axis: $V = \int_a^b 2\pi x [f(x) - g(x)] dx$

The Natural Logarithm Function: $\ln x = \int_1^x \frac{1}{t} dt$

Differentiation of the Natural Log: $\frac{d}{dx} [\ln g(x)] = \frac{1}{g(x)} \cdot \frac{d}{dx} g(x) = \frac{g'(x)}{g(x)}$

Integrating the Trig Functions using the Natural Logarithm Function:

$$\begin{aligned} \int \tan u du &= -\ln|\cos u| + c = \ln|\sec u| + c \\ \int \cot u du &= \ln|\sin u| + c = -\ln|\csc u| + c \\ \int \sec u du &= \ln|\sec u + \tan u| + c \\ \int \csc u du &= -\ln|\csc u + \cot u| + c = \ln|\csc u - \cot u| + c \end{aligned}$$

Derivatives of Inverse Trig Functions:

$$\begin{aligned} \frac{d}{dx} \sin^{-1} u &= \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx} & \frac{d}{dx} \cot^{-1} u &= -\frac{1}{1+u^2} \cdot \frac{du}{dx} \\ \frac{d}{dx} \cos^{-1} u &= -\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx} & \frac{d}{dx} \csc^{-1} u &= -\frac{1}{|u|\sqrt{u^2-1}} \cdot \frac{du}{dx} \\ \frac{d}{dx} \tan^{-1} u &= \frac{1}{1+u^2} \cdot \frac{du}{dx} & \frac{d}{dx} \sec^{-1} u &= \frac{1}{|u|\sqrt{u^2-1}} \cdot \frac{du}{dx} \end{aligned}$$

Integrals leading to Inverse Trig Functions:

$$\int \frac{du}{1+u^2} = \tan^{-1} u + c \quad \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + c \quad \int \frac{du}{u\sqrt{u^2-1}} = \sec^{-1} |u| + c$$

The Exponential Function: $\log_e x = \ln x$

If $y = \ln x$, then $e^y = x$

Integration by parts: $\int f(x)g'(x) dx = f(x)g(x) - \int f'(x)g(x) dx$

$$\frac{d}{dx} e^u = e^u \frac{du}{dx} \quad \int e^{ax} du = \frac{1}{a} e^{ax} + c \quad \int e^u du = e^u + c \quad \int a^u du = \frac{a^u}{\ln a} + c$$

Integrals Involving Trigonometric Powers:

$$\begin{aligned} \int \sin^n u du &= -\frac{\sin^{n-1} u \cos u}{n} + \frac{n-1}{n} \int \sin^{n-2} u du & \int \tan^n u du &= \frac{\tan^{n-1} u}{n-1} - \int \tan^{n-2} u du & \int \sec^n u du &= \frac{\sec^{n-1} u \sin u}{n-1} - \frac{n-2}{n-1} \int \sec^{n-2} u du \\ \int \cos^n u du &= \frac{\cos^{n-1} u \sin u}{n} + \frac{n-1}{n} \int \cos^{n-2} u du & \int \cot^n u du &= -\frac{\cot^{n-1} u}{n-1} - \int \cot^{n-2} u du & \int \csc^n u du &= -\frac{\csc^{n-1} u \cos u}{n-1} - \frac{n-2}{n-1} \int \csc^{n-2} u du \\ \int (\cos^m ax)(\sin^n ax) dx &= \frac{\cos^{m-1} ax \sin^{n+1} ax}{(m+n)a} + \frac{m-1}{m+n} \int (\cos^{m-2} ax)(\sin^n ax) dx & &= -\frac{\sin^{n-1} ax \cos^{m+1} ax}{(m+n)a} + \frac{n-1}{m+n} \int (\cos^m ax)(\sin^{n-2} ax) dx \\ \int \frac{\cos^m ax}{\sin^n ax} dx &= -\frac{\cos^{m+1} ax}{(n-1)a \sin^{n-1} ax} - \frac{m-n+2}{n-1} \int \frac{\cos^m ax}{\sin^{n-2} ax} dx & &= \frac{\cos^{m-1} ax}{a(m-n) \sin^{n-1} ax} + \frac{m-1}{m-n} \int \frac{\cos^{m-2} ax}{\sin^n ax} dx \\ \int \frac{\sin^m ax}{\cos^n ax} dx &= \frac{\sin^{m+1} ax}{a(n-1) \cos^{n-1} ax} - \frac{m-n+2}{n-1} \int \frac{\sin^m ax}{\cos^{n-2} ax} dx & &= -\frac{\sin^{m-1} ax}{a(m-n) \cos^{n-1} ax} + \frac{m-1}{m-n} \int \frac{\sin^{m-2} ax}{\cos^n ax} dx \end{aligned}$$

Basic Trigonometric Identities:

$$\begin{aligned} \cos^2 x + \sin^2 x &= 1 & \sin^2 x &= \frac{1 - \cos 2x}{2} & \sin 2x &= 2 \sin x \cos x \\ \tan^2 x + 1 &= \sec^2 x & \cos^2 x &= \frac{1 + \cos 2x}{2} & \cos 2x &= \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - \sin^2 x \\ \cot^2 x + 1 &= \csc^2 x & & & & \end{aligned}$$